## Grade 7/8 Math Circles

## February 20/21/22/23, 2023

## Math in Musical Scales - Problem Set

1. Complete the following operations. Reduce all fractions to simplest form. (As a challenge, solve without a calculator).
a) $\frac{24}{25} \div \frac{1}{2}$
b) $\frac{7}{10} \times \frac{3}{16}$
c) $\frac{5}{9} \div \frac{37}{40}$
d) $\frac{1}{3} \times \frac{9}{7}$
e) $\frac{6}{13} \div 2$
f) $\frac{25}{17} \times \frac{12}{5}$
2. Determine whether the following notes are a tone or a semitone apart. Looking at the keys on a piano may help for this question.
a) C and D
b) B and C
c) $C \sharp$ and $E b$
d) $\mathrm{E} \sharp$ and G
e) $G \sharp$ and $A \sharp$
f) $B$ and $A \sharp$
3. Find the missing values of the just scale in the table below.

| do | re | mi | fa | so | la | ti | do |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\frac{9}{8}$ | $\frac{5}{4}$ | $\frac{4}{3}$ | $\frac{3}{2}$ | $\frac{5}{3}$ | $\frac{15}{8}$ | 2 |
| 150 Hz |  |  |  |  |  |  | 300 Hz |

4. Find the missing values of the Pythagorean scale in the table below.

| do | re | mi | fa | so | la | ti | do |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\frac{9}{8}$ | $\frac{81}{64}$ | $\frac{4}{3}$ | $\frac{3}{2}$ | $\frac{27}{16}$ | $\frac{243}{128}$ | 2 |
| 200 Hz |  |  |  |  |  |  | 400 Hz |

5. Transpose the following Pythagorean scale with a tonic of $B b$ to have a tonic of F. Find all of the notes for the new scale.

| $\mathbf{B} b$ | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{E b}$ | $\mathbf{F}$ | $\mathbf{G}$ | $\mathbf{A}$ | $\mathbf{B} b$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\frac{9}{8}$ | $\frac{81}{64}$ | $\frac{4}{3}$ | $\frac{3}{2}$ | $\frac{27}{16}$ | $\frac{243}{128}$ | 2 |
| 58 Hz |  |  |  |  |  |  | 116 Hz |

6. Determine whether the following scale is in the Pythagorean tuning system or the just intonation system. (Hint: look at the intervals).

| do | re | $\mathbf{m i}$ | fa | so | la | ti | do |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50 Hz | 56.25 Hz | 62.5 Hz | $66 \frac{2}{3} \mathrm{~Hz}$ | 75 Hz | $83 \frac{1}{3} \mathrm{~Hz}$ | 93.75 Hz | 100 Hz |

7. Describe the benefits and drawbacks of Pythagorean tuning and just intonation.
8. If you wanted to transpose a song over and over again, which tuning system would you choose? Why?
